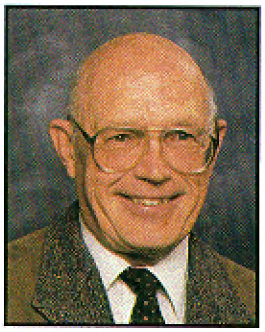
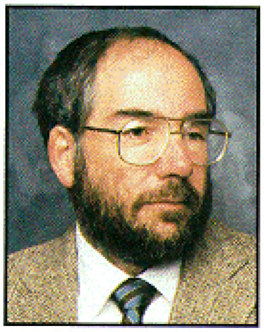
Root Locus and the analysis of rotor stability problems





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Since the invention of rotating machines, the pursuit of higher power output has driven machine speeds higher and higher. With the breaking of the first balance resonance "barrier" (achieved by DeLaval with a steam turbine in 1895), rotating machines were shown to be able to operate continuously above the first balance resonance. However, with this new capability came a new problem for machines using fluid-lubricated journal bearings: fluid-induced instability. Now recognized not only in bearings, but in seals and in the main flow of fluid-handling machines, fluid-induced instability has been a major focus of research efforts around the world as engineers have struggled to achieve higher operating speeds and higher power, while at the same time maintaining rotor stability. Over the years, many different methods and approaches have been developed by researchers to identify and understand the important parameters that influence rotor stability.

In Walter Evans' book, "Control-System Dynamics," [Ref. 3] is a powerful tool for the visualization and analysis of rotor stability: the Root Locus. Don Bently worked with Walter Evans at the time that this book was first published. The Root Locus technique is a powerful, graphically-based method for presenting rotor stability information, and, with the arrival of more powerful personal computers, is becoming increasingly easy to implement. The Root Locus is based on very simple concepts and can be easily understood and used for visualizing rotor system instability and parameter changes to enhance the stability margin.

The Root Locus

Use of the Root Locus technique requires a mathematical model of the rotor system under study. Although complicated, multi-Degree-of-Freedom models can be useful for the study of particular rotor system stability problems, a simple rotor model can provide a great deal of insight into the behavior of more complicated systems. The simple model presented in [Ref.5] will be used here to initially demonstrate the Root Locus procedure. The model leads to the 2nd-order, linear, differential equation,

$$M\ddot{z} + D\dot{z} + (K - jD\lambda\Omega)z = 0 \qquad (1)$$

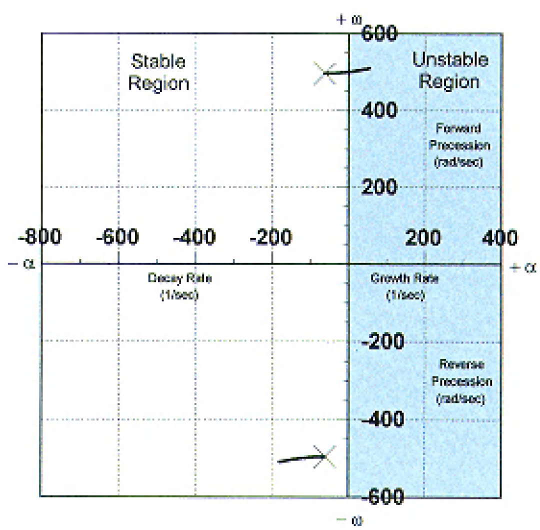

Figure 1

s-plane graph of the root loci of the two roots of the simple rotor model. These plots were made by varying rotor rotative speed, Ω, from 0 to 2000 rad/sec. The zero speed points are marked with an "X." The vertical axis marks the Threshold of Stability. The left half-plane corresponds to stable roots; the right half-plane corresponds to unstable roots for the system under consideration. The lower root poses no threat to stability, but the upper root crosses the vertical axis and moves to the right half-plane of instability.

where z = $x+jy$ is the complex displacement coordinate,

M = rotor system modal mass,

D = rotor system modal damping,

K = rotor system modal stiffness,

λ = fluid average circumferential velocity ratio,

Ω = rotor rotative speed,

j = $\sqrt{-1}$

and the dots indicate derivatives with respect to time, t. Note that x and y are the displacement coordinates in the plane perpendicular to the rotor axis of rotation.

In Root Locus analysis, the external forcing of the system is assumed to be zero in order to allow examination of the free vibration characteristics of the system. A standard solution technique for differential equations yields the *Characteristic Equation* of the system,

$$Ms^2 + Ds + K - jD\lambda\Omega = 0 \qquad (2)$$

Because Equation (2) is quadratic, there will be two roots. Each of the two roots of this equation are complex numbers of the form,

$$s = \alpha + j\omega \qquad (3)$$

where α and ω are complicated functions of M, D, λ, K, and Ω.

Each of the two roots has a direct and quadrature part. (While the direct and quadrature parts of the roots correspond to the "Real" and "Imaginary" parts of the roots as typically described in mathematical theory, there is nothing "imaginary" about the meaning of these equations.) The two roots lead to a general solution. For purposes of illustration, using only one root, the behavior of the rotor model can be expressed as

$$z = Ze^{\alpha t}e^{j\omega t} \qquad (4)$$

where Z is a complex rotor position constant which is not important in the Root Locus stability analysis.

The rotor natural frequency (the angular velocity of precession) is defined by ω, which, while a function of the rotative speed, Ω, is usually not equal to the rotative speed.

The rate of growth or decay of the amplitude of the orbit of precession, z, is controlled by α. It controls the decay rate or growth rate of the rotor orbit indirectly, through the first exponential function in Equation (4) and thus controls the transient response amplitude of the rotor system due to some disturbance. If $\alpha<0$, the first exponential function will become smaller with time, and the precession orbit will decay with time back to the original equilibrium position. However, if $\alpha>0$, the precession orbit will *increase* in size in an unbounded fashion[1].

The roots of the characteristic equation completely define the two time-dependent characteristics, growth (or decay) and precession frequency, of the free vibration of the rotor system. The two components, α and ω, of each root, s, can be graphed in the s-plane. By systematically varying a single rotor model parameter, curves in the s-plane can be generated. Each curve is a locus of roots of the characteristic equation, or a *Root Locus* curve and is a function of the chosen parameter.

Unbounded growth of the rotor orbit is the condition of rotor instability. Thus, for stability, all $\alpha<0$; that is, *all roots of the characteristic equation must lie in the left half of the s-plane*. The condition when any $\alpha=0$ defines the *Threshold of Stability*.

Such a pair of curves for the two roots of the rotor model are shown in Figure 1. These Root Locus curves were generated by varying the rotative speed, Ω, from 0 to 2000 rad/sec (in the counterclockwise or positive angular direction). The large "X" on the plots marks the zero rotative speed point. The roots are plotted with the Growth or Decay parameter, α, as the horizontal axis, and the natural frequency or precession rate, ω, on the vertical axis. Note that one of the roots is above the horizontal

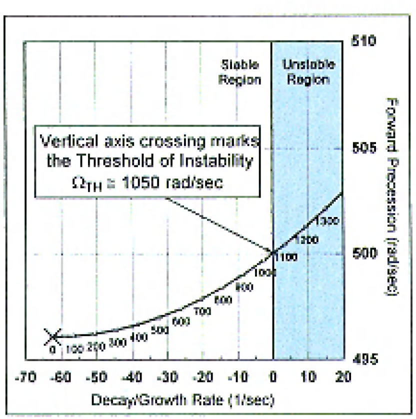

Figure 2

Important stability root of Figure 1. Rotative speeds are shown in rad/sec. The Threshold of Stability is shown to be near 1050 rad/sec.

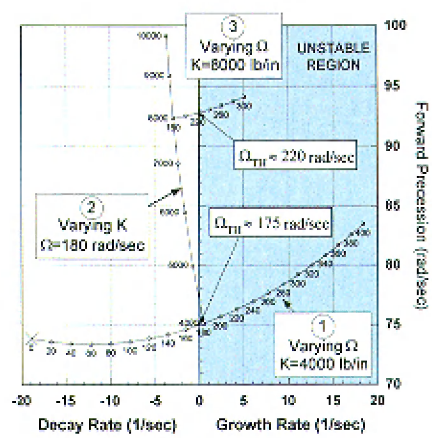

Figure 3

Example of a Root Locus analysis of a rotor system different than the one presented in Figures 1 and 2. Increasing the direct stiffness, K, from 4000 lb/in to 8000 lb/in results in an increase in the Threshold of Stability of more than 25%.

[1] *In real systems or more realistic models, of course, the orbit will not increase in size forever. The rotor will move into an operating region of higher eccentricity where the stiffness and damping become highly non-linear in behavior. This non-linearity invalidates the linear force model used in this analysis, and the rotor system settles into a new, stable limit cycle of whirl or whip.*

axis, indicating forward precession, while the other root is below the horizontal axis, indicating reverse precession. Note also that the reverse precession root moves toward greater decay rate with increasing rotative speed while the forward precession root moves toward lower decay rate (lower absolute value of α) with increasing rotative speed.

Most importantly, the forward root does cross the vertical axis at some speed. This indicates that, as α becomes equal to and then greater than zero, the rotor system will become unstable and proceed into a new limit cycle of either whirl or whip. In this model, the reverse root will never pose a threat to stability for positive, counterclockwise rotor rotation; it *would* pose a threat to stability for negative, clockwise rotor rotation. For that reason, *all* roots of a rotor model should be examined for potential zero crossings.

A graph showing only the important stability root of Figure 1 is presented in Figure 2. In this figure, the numbers along the curve represent rotor rotative speeds in rad/sec. The Threshold of Stability is shown to be approximately 1050 rad/sec.

An example of a Root Locus analysis is shown in Figure 3. In this example, a different rotor model than the one presented in Figures 1 and 2 was used. The rotor rotative speed was varied from 0 to 400 rad/sec. The Threshold of Stability for this rotor system was found to be about 175 rad/sec. At that point, the rotative speed was fixed at 180 rad/sec and the direct stiffness, K, was varied from its original value of 4000 lb/in to a final value of 10000 lb/in. The improvement in rotor stability can be clearly seen as the Root Locus crosses back into the left-hand side (the stable side) of the s-plane.

Note also that the rotor natural frequency (precession frequency) increases, as would be expected from an increase in stiffness of the system. Finally, the direct stiffness was fixed at 8000 lb/in, and the rotor rotative speed was increased from 180 rad/sec to 300 rad/sec. The new Threshold of Stability was found to be about 220 rad/sec, a 25% increase. This could reflect the improvement due to increasing bearing fluid pressure or to a deliberate, "friendly" misalignment placing the shaft into a higher eccentricity operating position.

A final example is a Root Locus analysis of a two-stage compressor. The compressor rotor was supported by two fluid-lubricated bearings and had a seal located at the mid-span between the two stages. The compressor had a history of whirling under certain startup conditions at a speed between the first and second lateral balance resonances. It was decided to compare the relative effectiveness of anti-swirl injection (see Ref. [10]) at the center seal to anti-swirl injection at the fluid-lubricated bearing.

A more complicated model was developed for this problem which resulted in a 6th-order Characteristic Equation with complex coefficients. The roots of this equation were found numerically using a Siljak polynomial method.

Since the characteristic equation was 6th-order, there were six roots. An s-plane Root Locus plot of the four rightmost roots of the model is presented in Figure 4. The other two roots were located far to the left in the s-plane and had no importance in the stability analysis. To create the Root Locus plots for this model, the shaft rotative speed was varied from 0 to 4000 rad/sec. Note that one of the roots crosses the vertical (precession) axis into the right, unstable half-plane. The crossing point defines the Threshold of Stability for this model. An enlarged view of this important stability root is shown in Figure 5. The rotor system is predicted to become unstable at a rotative speed of about 1050 rad/sec.

To examine the effectiveness of anti-swirl at the bearing, λ_C, the center seal

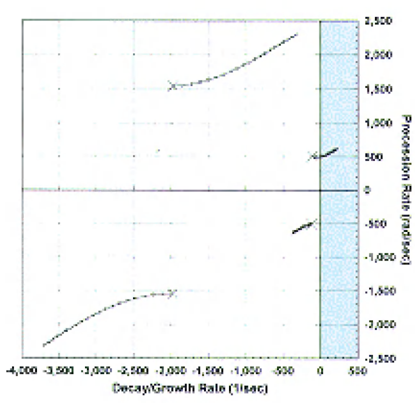

Figure 4

Root Locus s-plane plot of four of the six roots of the compressor model for rotative speed from 0 (marked with an "X") to 4000 rad/sec. The other two roots are located far to the left and do not have any importance in the stability analysis. One of the roots crosses the vertical axis into the right, unstable half-plane. The other roots pose no threat to stability in this speed range.

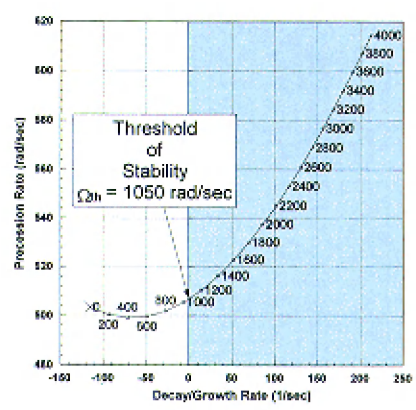

Figure 5

Enlarged view of the important stability root of Figure 4. The rotative speeds in rad/sec are indicated on the curve. The Threshold of Stability is approximately 1050 rad/sec. Note the similarity of this Root Locus to the one in Figure 2 which was generated using similar rotor parameters using the model of Equation (1).

average circumferential velocity ratio, was held constant at 0.48 while λ_B, the fluid-lubricated bearing average circumferential velocity ratio, was varied from 0.5 to 0.1 in five steps. To examine the effectiveness of anti-swirl at the center seal, λ_B was held constant at 0.48 while λ_C was varied from 0.5 to 0.1 in five steps.

Figures 6 and 7 show the effect of the two anti-swirl injection strategies on the important stability root of Figure 5. These Root Locus plots immediately reveal that anti-swirl is much more effective when applied to the center seal than when applied to the fluid-lubricated bearing. Even a small amount of anti-swirl injection at the seal results in a substantial improvement in the Threshold of Stability.

Conclusion

It is important to point out that the Threshold of Stability predicted by the Root Locus (and by any method of analysis, for that matter) is only as good as the model used in the analysis. Because of this, any stability analysis based on a model of a complex machine is unlikely to predict the *exact* Threshold of Stability.

The power of the Root Locus method comes from its ability to reveal how sensitive a rotor instability problem is to different rotor parameters and from its powerful visual presentation of those relationships. It allows one to consider various solution alternatives and look for the most cost-effective solution to a potential rotor stability problem.

Although complicated, multi-Degree-of-Freedom models can be useful for the study of particular rotor system stability problems (such as the compressor problem above), a simple rotor model can provide a great deal of insight into the behavior of more complicated systems. Comparison of Figure 5 (based on a complicated model) to Figure 2 (based on a simple model using similar rotor parameters) shows remarkable agreement in the predicted Threshold of Stability.

With the powerful symbolic and numeric desktop computation tools available today, the application of the Root Locus technique to rotor stability analysis has become much easier to implement. The power of this method makes it an excellent tool for engineers to use at both the design stage and for the exploration of potential fixes for stability problems encountered in the field.

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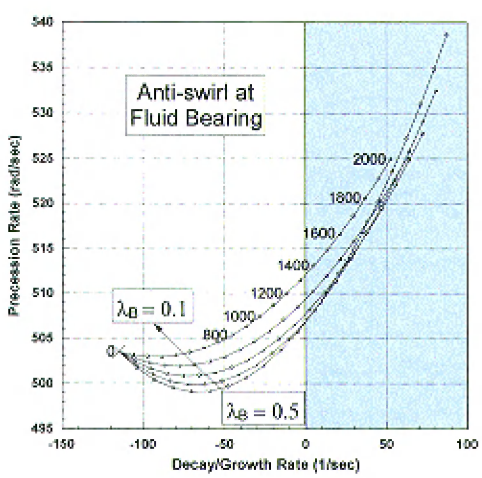

Figure 6

Effect of anti-swirl injection at the fluid-lubricated bearing on the stability of the compressor. For five values of λ_B (0.5 to 0.1), λ_C was held constant at 0.48, and rotor rotative speed was varied from 0 to 2000 rad/sec.

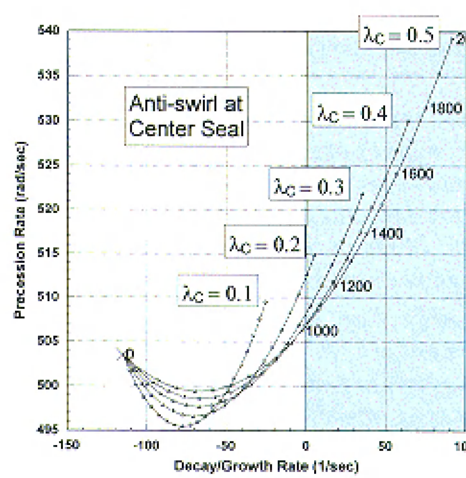

Figure 7

Effect of anti-swirl injection at the center seal on the stability of the compressor. For five values of λ_C (0.5 to 0.1), λ_B was held constant at 0.48, and rotor rotative speed was varied from 0 to 2000 rad/sec.